Technical Notes

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Mach Number Dependence of Laminar Near-Wake Transition for Slender Cones

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THE prediction of the complex transition process which the laminar near-wake flow undergoes before it becomes a fully turbulent flow has been an important concern for the re-entry physics community. There exists a large body of literature which encompasses experimental endeavors (primarily ballistic range experiments), some data correlations, and a few theoretical efforts. These efforts focused mainly upon determining the transition Reynolds number for a particular body shape at a variety of freestream Mach numbers ($M_\infty = 2$-20). In this Note we will show that the apparent Mach number dependence of the laminar wake transition process can be explained with the use of axis-based fluid properties and that a nearly universal axis-based transition Reynolds number independent of Mach number is found. The simple near-wake calculations described below are considered independent of downstream distance in the laminar run due to the lack of importance of laminar diffusion for high Reynolds number near wake transition (≤ 100 diameters). This feature allows for a straightforward correlation of the ballistic range data.

The most complete correlation study was published by Goldburg[1] in 1965. He obtained a prescription for a hypersonic wake transition map which included a far-wake and a near-wake description. The near-wake correlation provides a reasonable prediction of transition if the dimensional product of the ambient pressure and transition distance behind the body, $p_\infty x_{tr}$, is plotted vs a shoulder Mach number. Both sphere and cone data fell near his predicted line. Goldburg expected such a correlation (with shoulder Mach number) since the near wake should show a strong dependence on local hypersonic phenomena.[2,3] We will show that this flowfield dependence is very important; however, the Mach number dependence will be absorbed using some physical ideas related to the transition process. A more recent compilation of available transition data was provided by Waldbusser.[4]

The typical results of the experimental efforts were to show how laminar transition was delayed at higher M_∞. Zeiberg[5] demonstrates this point quite clearly. In his study, he shows (using available experimental data) that wedge, cone, sphere-cone, and sphere data can be correlated with M_∞ by multiplying the transition Reynolds number, $U_\infty x_{tr}/\nu_\infty$ by a bluntness factor $(M_\infty/M_{\rm edge})^2$. This "bluntness" Reynolds number still varies by three orders of magnitude. If we eliminate the bluntness effect and primarily concentrate on slender cones, this same Mach number dependence remains. Is this compressibility effect fundamental? The correlation method developed below attempts to answer this question.

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In order to limit our discussions, we consider laminar near-wake (0-100 diameters downstream) transition for sharp slender cones with half-angles between 6.3 and 12.5 degrees. The data used in the analysis come from Lyons, Brady, and Levensteins,[6] Levensteins and Krumins,[7] Slattery and Clay,[8] Wilson,[9,10] and Pallone, Erdos, and Eckerman,[11] – all ballistic range experiments. In Fig. 1 (open symbols) we shown the transition data plotted in typical transition coordinates, i.e., $R_{\infty,tr} = U_\infty x_{tr}/\nu_\infty$ vs. M_∞. Notice the aforementioned dramatic change in $R_{\infty,tr}$ with M_∞.

In considering the Mach number dependence of transition, it is clear that $R_{\infty,tr}$ is not an ideal measure of the nonlinear transition process. First, the wake development is governed by the velocity difference across the wake, $\Delta U_{\rm axis} = U_\infty - U_0$, rather than the freestream speed U_∞. Second, the relevant gas properties should be those within the viscous wake rather than properties in the freestream or at the wake edge. Many experiments have shown that linear instability begins at the location of maximum shear and then spreads to the rest of the wake. The subsequent development of nonlinear "turbulent-like" fluctuations is first established on the axis, as depicted graphically by Sato and Okada.[12] Hence, it seems appropriate to base our transition Reynolds number on the viscosity at the axis or at the maximum shear location. We have somewhat arbitrarily chosen axis conditions to be the more appropriate; the temperature difference between the axis and peak shear

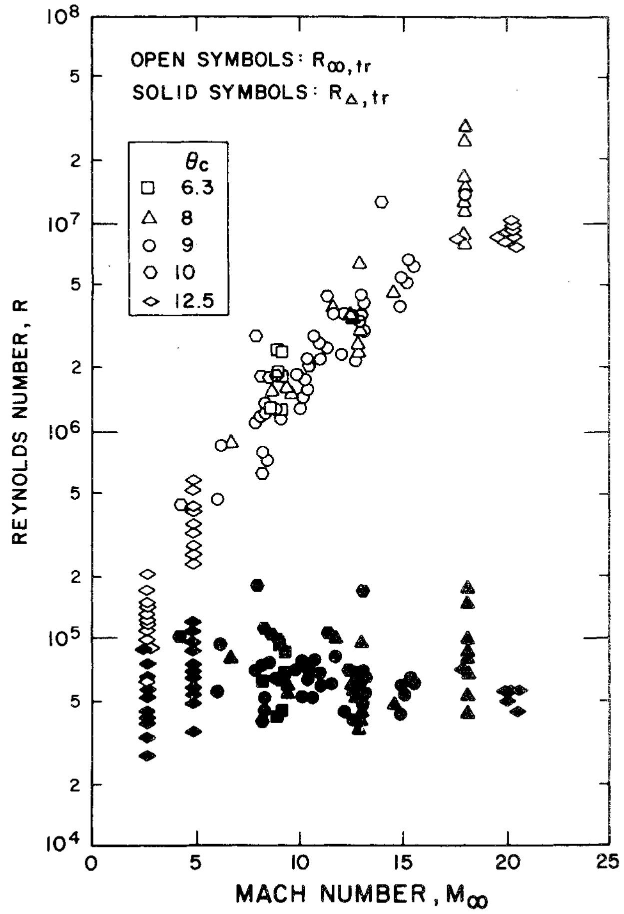

Fig. 1 Laminar near-wake transition for slender cones.

location is not great, of course, and either temperature would differ greatly from the edge temperature at high Mach numbers. (Previous attempts[11,13,14] to utilize nonfreestream parameters have been limited to edge values.)

The transition Reynolds number is now defined as $R_{\Delta,tr} = \Delta U_{\text{axis}}\, x_{tr}/\nu_{\text{axis}}$. If we replot the transition data base using $R_{\Delta,tr}$ we obtain the solid symbols of Fig. 1. The same data are remarkably free of Mach number dependence. An average $R_{\Delta,tr} \approx 8\times10^4$ can be chosen from the plot. Lines drawn a factor of two above and below 80,000 bracket the bulk of these data. This factor of two is a very reasonable one when dealing with the prediction of transition processes. The conclusion is obvious. A supersonic ($M_\infty \geq 2$) laminar wake will undergo transition in the near wake of a slender cone at a universal Reynolds number $\approx$80,000. Two aspects of this correlation remain to be described. The approximate method used to compute the axis properties must be detailed, and the reasons for the data collapse require explanation.

We now describe the calculation of ΔU_{axis}, which is the axial near-wake velocity defect after pressure relaxation has occurred but before momentum diffusion occurs, and ν_{axis}. Clearly, both the defect and the axis viscosity change in a manner to reduce the infinity-based Reynolds number. A detailed flowfield solution for the boundary layer and the base region is required for the proper determination of the local near-wake properties. This would be extremely laborious for the wide range of conditions spanned by the data. In light of this difficulty, we made certain approximations, which in view of the inherent randomness in the occurrence of transition seem reasonable for present purposes. Each transition data point was analyzed with the method described below.

A major simplification is possible because laminar diffusion is slow downstream of the base region, under conditions of present interest. Thus near-wake axis properties may be considered independent of distance during the laminar run. In fact the lack of importance of laminar diffusion is probably the feature which essentially distinguishes high Reynolds number near-wake transition from low Reynolds number far-wake transition. Laminar diffusion will be negligible when the diffusion time D_w^2/ν_{axis} is much greater than the transit time $x/\Delta U_{\text{axis}}$. Or rewritten with $D_w \sim D$ and x associated with x_{tr}

$$\frac{(\Delta U_{\text{axis}} x_{tr})/\nu_{\text{axis}}}{(x_{tr}/D)^2} \gg 1 \qquad (1)$$

Our correlation study found that $\Delta U_{\text{axis}} x_{tr}/\nu_{\text{axis}} \cong 8\times10^4$. Hence, the ratio in Eq. (1) is equal to 8 when $x_{tr}/D = 10^2$, and equal to 800 when $x_{tr}/D = 10$. Consequently, laminar diffusion will be small in our range ($\leq$100 diameters) of interest.

To approximate the near-wake axis temperature (to compute ν_{axis}), we accounted for the effects of viscous heating in the boundary layer and the expansion from the cone sidewall pressure to ambient pressure. We started with the peak boundary-layer temperature, which should be an appropriate typical temperature for the inner boundary-layer streamlines. For a perfect gas the Crocco integral relation may be used to show that

$$\frac{T_{\text{max, b.l.}}}{T_\infty} = \tfrac{1}{4} + \tfrac{1}{4}\left(1 + \frac{\gamma-1}{2}M_\infty^2\right) + \tfrac{1}{2}\frac{T_w}{T_\infty} + \frac{1}{2(\gamma-1)M_\infty^2}\left(1 - \frac{T_w}{T_\infty}\right)^2 \qquad (2)$$

The wall temperature T_w was taken to be approximately T_∞ for all cases. Then, to obtain the near-wake temperature the isentropic expansion formula gives

$$T_{\text{axis}} = T_{\text{max}}\left(\frac{p_c}{p_\infty}\right)^{-(\gamma-1)/\gamma} \qquad (3)$$

where p_c is the cone pressure. The axis viscosity follows from the Sutherland law ($\nu_{\text{axis}} \sim T_{\text{axis}}^{1.6}$ is a good approximation). In this scheme, the effects of chemistry (dissociation and recombination) and near-wake viscous and shock heating are neglected. These effects go in opposite directions and hence tend to cancel. Also, as detailed above, diffusion becomes important for $x/D \sim 100$, which is beyond the range of present interest.

The axis velocity difference ΔU_{axis} is much less sensitive to Mach number than is the axis temperature. Conservation of mass and momentum may be used to determine ΔU_{axis}. Assuming the near-wake (radial) velocity profile to be universal in density-transformed coordinates, the ratio of momentum to mass flux gives the following expression

$$\frac{\Delta U_{\text{axis}}}{U_\infty} = \alpha\left\{\frac{C_{D_f} + C_{D_p}}{2\dot{m}_{\text{b.l.}}}\right\} \qquad (4)$$

Where C_{D_f} is the boundary-layer skin-friction coefficient, C_{D_p} the base-pressure drag coefficient, $\dot{m}_{\text{b.l.}}$ the boundary-layer mass flow rate normalized by $\rho_\infty U_\infty \pi R_c^2$, and α is a factor involving the integral of the velocity profile. Equation (4) is simply a statement that the wake contains the viscous drag plus the base drag of the body. For α we use 3, a value appropriate to a squared parabola or a Gaussian velocity profile. The quantities C_{D_f} and $\dot{m}_{\text{b.l.}}$ can be obtained from numerous studies (e.g., Kohrs et al.[15]). At hypersonic speeds ($M \geq 10$) the base pressure term in Eq. (4) is negligible, and the velocity difference is insensitive to Mach number. For lower Mach numbers, Waldbusser's[16] extensive compilation of base-flow data was used to estimate C_{D_p}. Typically, $\Delta U_{\text{axis}}/U_\infty$ is estimated to be 0.21 at $M_\infty = 20$ and 0.48 at $M_\infty = 5$.

To summarize, we examine the reasons why our more natural transition Reynolds number, $R_{\Delta,tr}$, collapses the data. It is primarily due to the strong ($\sim$1.6 power) dependence of ν_{axis} on T_{axis}. And since the axis temperature changes approximately with the square of the freestream Mach number, the rapid rise of $R_{\Delta,tr}$ with M_∞ can be explained by the increase in the viscosity of the hot wake core. The elevated axis viscosity is the major compressibility effect on wake transition. The axis velocity defect does not vary much with M_∞ and thereby changes the axis viscosity-based correlation only slightly; that is, it is changed just enough to remove the Mach number dependence entirely.

Based upon this approximate but consistant correlation approach, we would expect the above interpretation to also be valid for blunted slender as well as spherical type shapes. Furthermore, this study implies that a compressible (supersonic) viscous stability calculation should predict the critical Reynolds number based upon axis properties to be a weak function of Mach number M_∞. Such calculations utilizing the parallel-flow approximation should be rather accurate due to the very slow decay of the laminar near wake.

Acknowledgment

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Deformations and Thermal Conductances of Cone to Flat Contacts

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Introduction

IN many engineering situations it is important to control and to be able to predict the thermal conductance of metallic joints. This is often difficult as there are several interrelated parameters which cannot be specified accurately. One of the main requirements is a knowledge of the contact geometry, viz. the sizes and shapes of the small actual contact spots, and how these are distributed over the apparent contact zone. Mathematical models of the contact zone usually specify a contact spot, for which the potential heat flowfield may be determined. The main purpose of this Note is to compare the predicted and measured thermal conductances of single contact spots of easily controlled shapes, viz. cones pressing against plane surfaces, for which the actual contact areas may be measured accurately. Another purpose is the determination of actual support pressure of cone to flat contacts over a wide range of cone angles, as existing data are incomplete.

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Literature Survey

Geometrically simple models of contacts have been examined mathematically and physically in several studies of thermal conductance, e.g. Refs. 1-3, and now form the main bases of computer-aided predictions of the thermal behavior of metallic joints. In all these predictions it is necessary to specify an appropriate "hardness" (or true contact pressure) of the materials in contact, accounting for both elastic and plastic modes of deformation, and peculiarities of shape. The heat-transfer tests and contact area measurements reported in Ref. 4 were sufficiently anomalous to provoke the investigation reported in this paper, using very simple shapes of contact elements, and joint pairs of the same material.

Theory

Cone Deformation

Although the plastic behavior of metallic cone to flat contacts under compression has been studied since 1926,[5,6] there is still no general analytical solution available for the prediction of the shape of a cone being flattened against a plane. However, the behavior of the two-dimensional wedge of rigid-plastic material deforming against a smooth plane is well established,[6] and is used herein as the basis of an approximate slip-line field solution for the cone problem, from which the deformed shape and the true contact pressure may be determined. Observations made during deformation tests indicated that the predicted and measured shapes matched closely.

Referring to Fig. 1, assuming incompressible material, and a deformation mode in which lengths m and n are equal, the relationship between the cone semiangles β and the slip angle α can be shown to be

$$\tan\beta = \frac{(1+\sin\alpha)^3}{\cos\alpha\,(3+3\sin\alpha+\sin^2\alpha)}$$

This equation is shown plotted on Fig. 2, which also shows measured angles of cones of mild steel and of aluminium deforming against a smooth hard surface (made of tool steel). For large cone angles, α is almost equal to β.

Thermal Conductance

The temperature distribution in a homogeneous and isotropic conductor during steady-state heat flow conditions is described by Laplace's equation. For axisymmetric

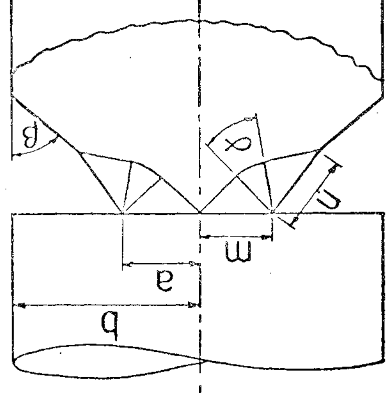

Fig. 1 Cone to flat joint showing slip-line field in deforming cone.